

# ASReml tutorial

## D1 GLM & GLMM

Arthur Gilmour





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# Generalized Linear Mixed models

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- Iteratively reweighted least squares
- Data types in exponential family
  - Binary [0,1]
  - Binomial [r,n]
  - Poisson
  - Gamma
  - Negative Binomial
- Variance is dependent on the mean



# Method

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- Analyse on a transformed scale using Link function and distribution specific weights

$$\mathbf{y} = \mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\mathbf{u} + \mathbf{D}^{-1}(\mathbf{O} - \mathbf{E})$$

where  $\mathbf{D}$  is derivative of link function

$\mathbf{O}$  is the observed value

and  $\mathbf{E}$  is  $g(\mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\mathbf{u})$ , the expected value  
weights are a function of  $\mathbf{D}$  and  $\mathbf{E}$

# Method continued

- Apply usual REML to this working variable  $y$  updating  $E$  and  $D$ , weights and  $y$  each iteration.

- Binomial Logit

$$\mathbf{Y} = \mathbf{X}\hat{\boldsymbol{\tau}} + \mathbf{Z}\tilde{\mathbf{u}}$$

$$\hat{E} = p = 1/(1 + e^{-Y}) \rightarrow Y = \log(p/(1 - p))$$

Derivative is  $p(1 - p)$

Binomial weight is  $np(1 - p)$

Analysis weight is  $n/[p(1 - p)]$



# Analysis of Deviance

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- Without random effects, this method is standard GLM (Generalized linear model) and ASReml also calculates the -2L, the maximum likelihood deviance, used to assess improvement of fit from adding an extra term to the model (Analysis of Deviance).



# GLMM Disclaimers

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- With random effects, the 'Deviance' is not valid for testing changes in the fixed model,
- the REML LogL reported cannot be used to test changes to the random model because  $y$ , the working variable changes between models.



# GLMM Disclaimers

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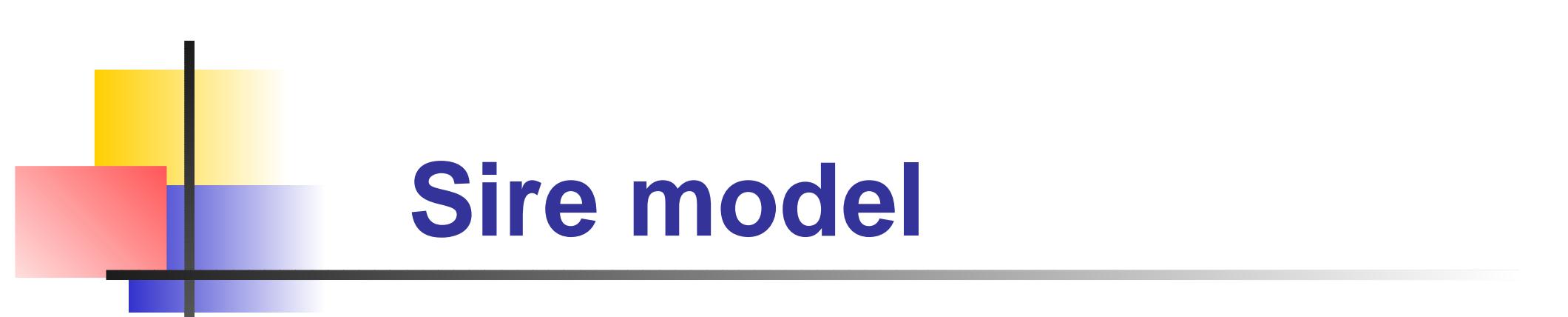
- The Method does not work for Animal Model variance components: it is biased - especially when number of random effects exceeds N/10.
- In most cases, effectively equivalent results are obtained from analysis on the observed scale



# Families and Links

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- !BIN [!ID !LOGIT !PROBIT !COMP] !TOTAL  $n$   
  !POIS [!ID !LOG !SQRT]  
  !GAMMA [!ID !LOG !INV] [!phi  $p$ ]  
  !NEGBIN [!ID !LOG !INV] [!PHI  $p$ ]  
General: !OFFSET  $o$  !DISP !DEV !WORK  
  !RESP !PEAR



# Sire model

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- Binomial has reasonable base for genetics because the logit [probit] link functions imply underlying residual logistic [normal] distributions with error variance 3.3 [1]
- Can fit the model for Poisson but I am unaware of proper genetic basis for estimating heritability say.

# LAMB data

## ■ Foot shape Score

```
#Yr Grp Sex Sire xxx tot 15 14 ls lr  
# 1 1 1 1 18 39 33 6 6 1  
# 1 1 0 1 18 50 41 9 2 0  
# 1 1 1 2 18 35 30 4 1 0  
yr 2 Grp 5 Sex Sire 18  
xxx tot L5 !/tot  
L4 !/tot LS !/tot LR !/tot  
lamb.dat !SKIP 1  
L5 !BIN !LOG !TOTAL tot ~ mu ,  
Grp Sex Sex.Grp !r Sire
```



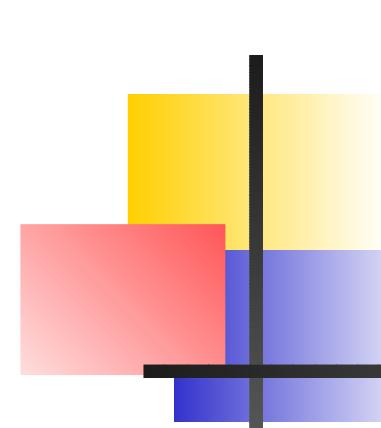
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## D2 ANOVA

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# The issues of ANOVA in REML

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- Fixed effects in SPARSE part of model
- Types of Sums of Squares
  - Incremental and Conditional (!FCON)
- Test statistic
- Denominator Degrees of Freedom
  - DDF - 1 suppressed
  - DDF 1 numerical derivative
  - DDF 2 algebraic derivative

# OATS - fixed ANOVA

## ■ !FCON

$Y \sim mu bl var bl.wp nit nit.var$   
ordered to get correct SS and DF.

BUT ASReml uses wrong F ratio because we have not specified that bl and wp are random.

## ■ Degrees Freedom Stratum Variances

45.00	177.083	1.0
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Source	Modterms	Gam	Component	C/SE	%C
Variance	72	45	1.0	177.083	4.74
				0P	

# Fixed ANOVA

AnOVar	Num	DenDF	F-inc	F-con	M	Prob
mu	1	45.0	4395.31	NA	.	NA
blocks	5	45.0	17.93	17.93	A	<.001
variety	2	45.0	5.04	5.04	a	0.011
bl.wp	10	45.0	3.40	3.40	B	0.002
nitrogen	3	45.0	37.69	37.69	A	<.001
nit.var	6	45.0	0.30	0.30	B	0.932

# Proper tests

- mu:  $F = 245.14 = 4395.31 / 17.31$  with 1,5 DF
- var:  $F = 1.49 = 5.04 / 3.40$  with 2,10 DF

■ mu	1	45.0	4395.31	NA	.	NA
blocks	5	45.0	17.93	17.93	A	<.001
variety	2	45.0	5.04	5.04	a	0.011
bl.wp	10	45.0	3.40	3.40	B	0.002

# OATS - REML analysis

## ■ !FCON

Y ~ mu nit var nit.var !r bl bl.wp

Degrees Freedom and Stratum Variances

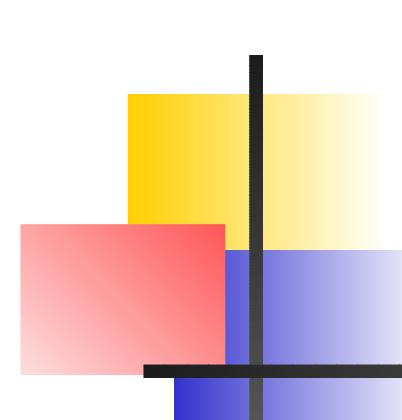
5.00	3175.06	12.0	4.0	1.0
10.00	601.331	0.0	4.0	1.0
45.00	177.083	0.0	0.0	1.0

Source	Modterms	Gamma	Component	/SE	%C
blocks	6	6	214.477	1.27	0P
bl.wp	18	18	106.062	1.56	0P
Variance	72	60	177.083	4.74	0P

# OATS - REML

- Proper F-inc value
- F-con is here same as F-inc because a balanced design.

AnOVar	Num	DenDF	F-inc	F-con	M	Prob
mu	1	5.0	245.14	NA	.	NA
nitrogen	3	45.0	37.69	37.69	A	<.001
variety	2	10.0	1.49	1.49	A	0.272
nit.var	6	45.0	0.30	0.30	B	0.932



# Wald tests

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- F-inc based on Sum of Squares explained by adding the term in the order specified divided by NumDF and  $\sigma^2$
- In mixed model, not easy to calculate DenDF.  
Not available in ASReml 1.1  
Not available for large models in ASReml 2  
Not necessarily the same for F-inc and F-con



# Order of fitting

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- Main effects before interactions:
  - If you specify an interaction first, the main effects will typically disappear
  - If you put an interaction in the sparse section, it will sweep out mu, and higher order terms.

# In practice

- Interpret the following ANOVA

Src	NDF	DDF	F-inc	F-con
x1	1	45	1.	21.
x2	1	45	23.	1.
x3	1	45	10.	10.

# In practice

- Interpret the following ANOVA

Src	NDF	DDF	F-inc	F-con
X1	1	45	1.	21.
X2	1	45	23.	1.
X3	1	45	10.	10.

- X1+X2+X3 explains 34.  
X1+X2 explains 24  
X1+X3 explains 33  
X2+X3 explains 13

# In practice

- Interpret the following ANOVA

Src	NDF	DDF	F-inc	F-con	M	%P
Region	2	45	11.	21.	b	
Site	11	45	8.	1.	A	
Var	12	45	10.	10.	A	
Var.Site	99	45	5.	5.	B	

# In practice

- Interpret the following ANOVA

Src	NDF	DDF	F-inc	F-con	M	%P
Region	2	45	11.	21.	b	
Site	11	45	8.	1.	A	
Var	12	45	10.	10.	A	
Var.Site	99	45	5.	5.	B	

- Sites are nested in Region so Region may not be tested after Site (Var.Site).

# Conditional F-tests



Term	F-inc	F-con
A	A	A B,C,B.C
B	B A	B A,C,A.C
A.B	A.B A,B	A.B A,B,C,A.C,B.C
C	C A,B,A.B	C A,B,A.B
A.C	A.C A,B,A.B,C	A.C A,B,C,A.B,B.C
B.C	B.C A,B,A.B,C,A.C	B.C A,B,C,A.B,A.C
A.B.C	A.B.C A,B,A.B,C,A.C,B.C	A.B.C A,B,A.B,C,A.C,B.C

# More complicated example

AnOVar	NumDF	DenDF	F-inc	F-con	M	Prob
FD0	1	220.0	36.81	34.30	C	<.001
GreenDM	1	10.7	9.52	8.59	A	0.014
Pos	1	10.0	93.09	93.03	A	<.001
Green.Pos	1	10.0	0.57	0.04	B	0.847
HM	1	9.5	0.12	3.39	B	0.095
Pos.HM	1	10.0	9.40	9.88	C	0.010
SLen	1	420.8	0.94	0.82	A	0.368
SL.Pos	1	1004.2	1.66	1.72	B	0.190
SL.Green	1	396.3	4.18	6.18	B	0.014
SL.Pos.Green	1	1275.0	1.55	11.52	C	<.001

# More complicated example

■ at(Group,4)	1	11.2	5.02	0.02	A	0.878
at(Group,1)	1	9.4	5.98	0.00	A	0.978
at(G,4).Pos	1	10.1	0.22	0.60	B	0.456
at(G,1).Pos	1	9.9	0.99	0.15	B	0.703
at(G,4).Pos.SL	1	1786.9	3.20	18.67	C	<.001
at(G,1).Pos.SL	1	1125.4	4.69	10.95	C	0.001
Clover	1	9.8	0.71	0.73	A	0.414
Clov.SL	1	535.4	1.07	1.07	B	0.302
Clov.Pos	1	10.0	0.57	0.57	B	0.467
Clov.SL.Pos	1	1269.5	16.06	16.06	C	<.001



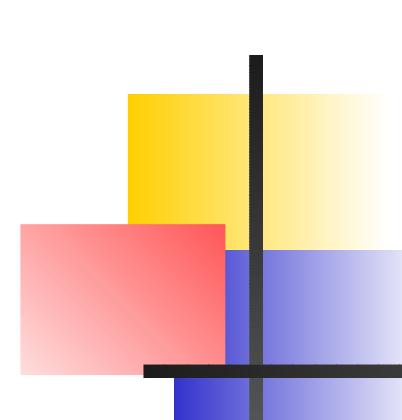
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## D3 Prediction in Linear Mixed Models

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# Prediction

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- This is the process of calculating linear combinations of the effects to summarise aspects of the analysis. e.g.
  - plotting a fitted spline curve - creating treatment means
- Details available in Welham et al and Gilmour et al. with coauthors B Cullis, B Gogel & R Thompson



# Plan

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- Introduction - The linear mixed model
- Prediction in Large problems
- Estimability
- Random effects
- Specification



# linear mixed model

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- $\mathbf{y} = \mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\mathbf{u} + \mathbf{e};$   
 $\mathbf{y} \sim \mathbf{N}(\mathbf{X}\boldsymbol{\tau}, \mathbf{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}^T);$   
 $\mathbf{u} \sim \mathbf{N}(0, \mathbf{G}); \mathbf{e} \sim \mathbf{N}(0, \mathbf{R})$

# Mixed model equations

- $$\begin{pmatrix} \mathbf{X}^T \mathbf{R}^{-1} \mathbf{X} & \mathbf{X}^T \mathbf{R}^{-1} \mathbf{Z} \\ \mathbf{Z}^T \mathbf{R}^{-1} \mathbf{X} & \mathbf{Z}^T \mathbf{R}^{-1} \mathbf{Z} + \mathbf{G}^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{\tau} \\ \mathbf{u} \end{pmatrix} = \begin{pmatrix} \mathbf{X}^T \mathbf{R}^{-1} \mathbf{y} \\ \mathbf{Z}^T \mathbf{R}^{-1} \mathbf{y} \end{pmatrix}$$
 rewritten as  $\mathbf{C}\boldsymbol{\beta} = \mathbf{W}^T \mathbf{R}^{-1} \mathbf{y}$   
giving  $\hat{\boldsymbol{\beta}} = \mathbf{C}^{-1} \mathbf{W}^T \mathbf{R}^{-1} \mathbf{y}$   
prediction  $\pi = \mathbf{D}\hat{\boldsymbol{\beta}}$   
with variance  $\mathbf{D}\mathbf{C}^{-1}\mathbf{D}^T$

# Large Models

- ASReml fits large models; avoids forming all elements of  $C^{-1}$

- Absorbing  $C$  in

$$\begin{pmatrix} \mathbf{y}^T \mathbf{R}^{-1} \mathbf{y} & 0 & \mathbf{y}^T \mathbf{R}^{-1} \mathbf{W} \\ 0 & 0 & \mathbf{D} \\ \mathbf{W}^T \mathbf{R}^{-1} \mathbf{y} & \mathbf{D}^T & \mathbf{C} \end{pmatrix} \text{ gives}$$
$$\begin{pmatrix} \mathbf{y}^T \mathbf{P} \mathbf{y} & -\boldsymbol{\pi}^T \\ -\boldsymbol{\pi} & -\mathbf{D} \mathbf{C}^{-1} \mathbf{D}^T \end{pmatrix}$$

- Done in final iteration

# Estimability

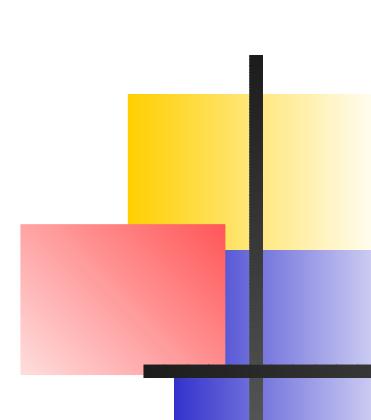
- No data. Height of males, females and angels
- Overparameterization: model  $\mu + \alpha_i$   
 $\alpha_i$  is non-estimable - infinitely many solutions

$\mu$	$\alpha_1$	$\alpha_2$	$\alpha_3$
0	10	12	14
10	0	2	4
12	-2	0	2

# Estimability

- Over-modelling  $\mu + \alpha_i + \beta_j$

12	10	14	*	*
11	13	15	*	*
*	*	*	17	18



# Averaging over incomplete tables

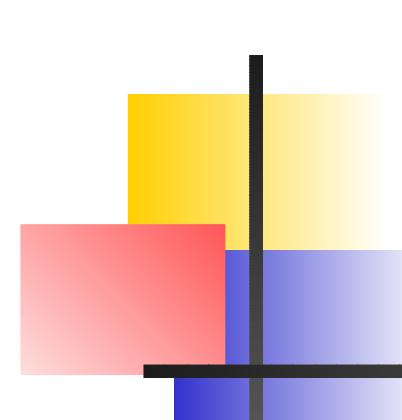
---

- No problem if table is all estimable even though some cells have no actual data i.e. interaction is omitted or is random
- Sometimes sensible to average only over cells containing data. E.g. variety.year.location table: form variety table averaging over the experiments which form an incomplete year.location table.

# Detection of Nonestimability

- During absorption process, nonestimability is present when the row of  $D$  is not zero but the row of  $C$  is zero.

-25/5		-1/5					
*		0	-1/0				
-30/6		0	0	-1/5			
<hr/>							
25		1	0	0		5	
0		0	1	0		0	0
30		0	0	1		0	0
							5



# Random effects

---

- The residual term is usually ignored. But kriging is prediction based on a correlated residual.
- Other random terms might be
  - error terms (usually ignored) or
  - treatment terms - may be predicted, averaged (conditional) or ignored (marginal).
- averaging over random lowers SE, may not affect SED



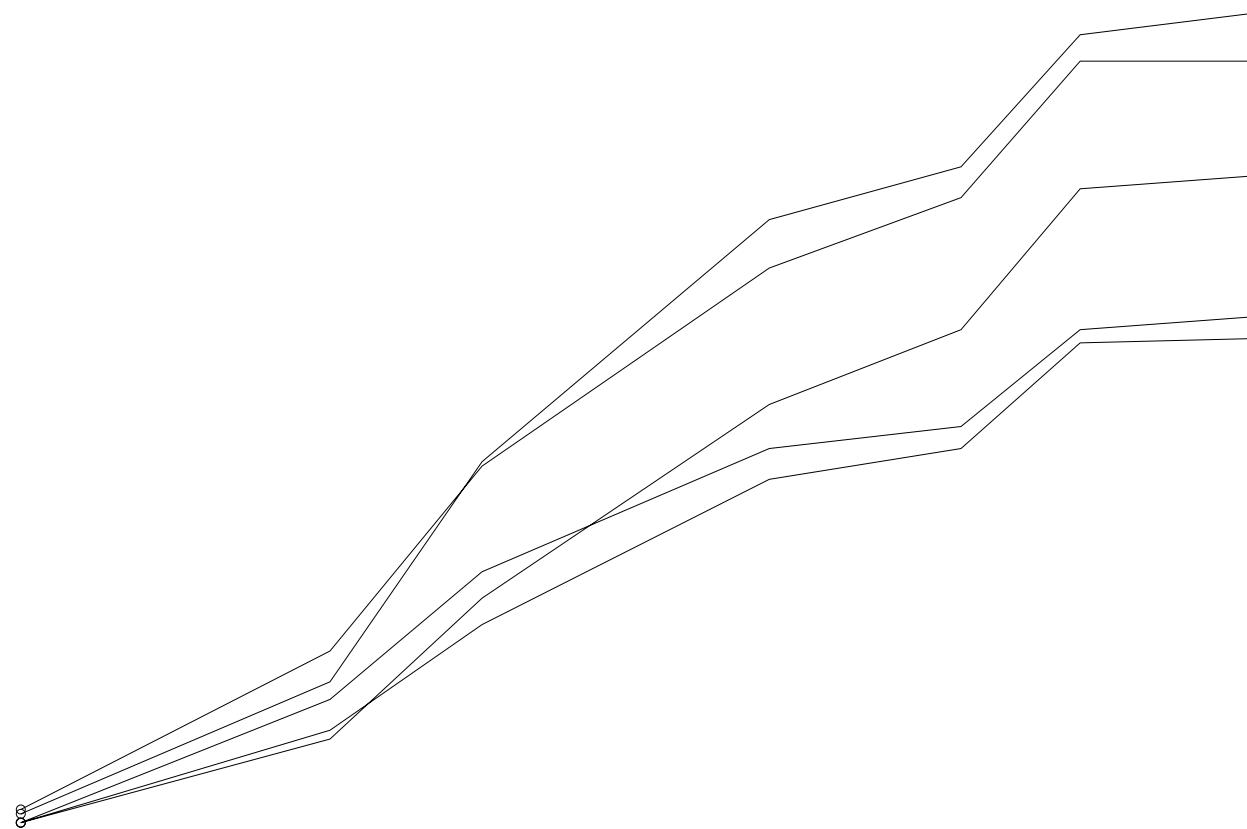
# Specification

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- Mixed model consists of model terms derived from factors and covariates.
  - Classify set - defines table to predict
  - Average set - usually other fixed variables
  - Ignored set - usually other random variables
- Set specific levels to be predicted
- Whether to average over cells with data
- Specific in/ex-clusion of model terms
- Whether to print nonestimable solutions

# Growth of 5 Orange trees

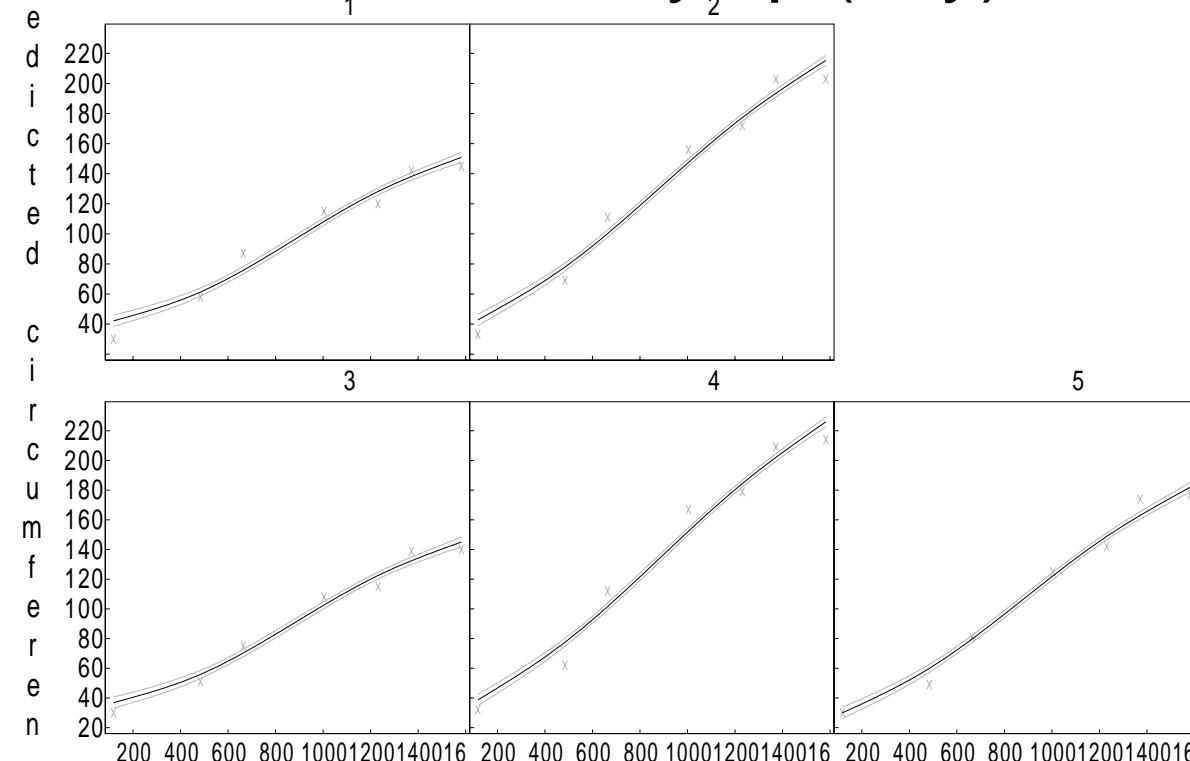
Y=circumference Title: orange.  
Y-axis: 30.0000 214.0000 X=age  
X-axis: 118.0000 1582.0000



# Orange tree model

- Fixed - mu season day
- Random - Tree Tree.day spl(day)

- 





# Conclusion A

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- Prediction from a linear model is essential for reporting results
- User must be advised if predictions are non-estimable functions
- Algorithm must be able to handle large and complex models
- Algorithm must give control of how to handle random terms



# Predict directive

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- basic PREDICT syntax
- Where to place the PREDICT directive
- additional qualifiers
- other issues - spline interpolation



# Basic syntax

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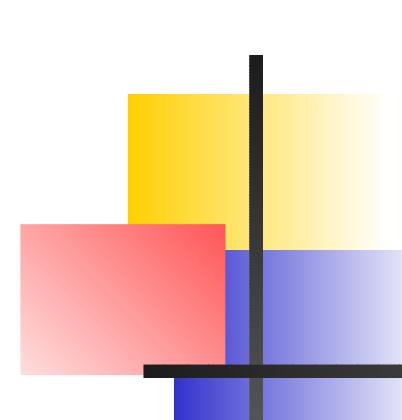
- PREDICT classify set [qualifiers]
  - !present present set,
  - !average factor [weights]
  - Each factor name in the classify and present sets may be followed by list of actual values to predict



# Where it goes

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- Immediately after the linear model (before the Variance header line)
  - or after all R and G structure lines
- There can be many PREDICT statements.
  - Parsed at start
  - Design formed before first iteration
  - Prediction done in final iteration



# Basic concepts

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- A hyper table defined by the factors in the model possibly ignoring some which only appear in random terms e.g. breed year animal sex
- Collapsed by averaging dimensions to produce the predict table.
- E.g    `y ~ mu variety !r block  
predict variety`



# Hypertable control

---

- **!PARALLEL a b c**
  - In model need to link factor and covariate versions of same data with lin() or spl()
- **data line qualifiers !PPOINTS and !PVALS for covariates (especially splines)**
- **specify particular levels of classify and present factors.**
  - Predict breed sex 1



# Fine control - ignoring model terms

- Default - ignore model terms involving factors which only occur in random model terms (e.g. animal but not spl())
- Control
  - !USE model\_terms
  - !ONLYUSE model\_terms
  - !IGNORE model\_terms
  - !EXCEPT model\_terms



# Output control

- !PRINTALL
- !SED, !VPV
  - increased memory required for V matrix
- Backtransformation !LOGIT !PROBIT  
!INVERSE !SQRT !COMPLOGLOG !LOGN  
!POWER p Deprecated qualifier
- !FITMARGIN if classify set is two-way table causes marginal tables to be produced.



# General

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- !FINAL command line option does one  
!CONTINUE iteration making it easy to do  
modelling then add predict statements at